DESY 97-203
HUB-EP-97/72

Lattice Renormalization of Quark Operators*

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We have technically improved the non-perturbative renormalization method, proposed by Martinelli et al., by using quark momentum sources and sinks. Composite two-fermion operators up to three derivatives have been measured for Wilson fermions and Sheikholeslami-Wohlert improved fermions in the quenched approximation. The calculations are performed in the Landau gauge on $16^3 32$ lattices at $\beta = 6.0$ for 3 κ values in each case. The improved sources greatly decrease the statistical noise. We extract and discuss here renormalization factors for local operators and moments of the structure functions for Wilson fermions.

1. INTRODUCTION

In this paper we want to discuss some general and technical aspects of calculating non-perturbatively renormalization factors of bilinear quark operators by imposing renormalization conditions on off-shell quark Green's functions on the lattice [1].

This method offers the possibility of computing non-perturbative contributions and all orders of QCD perturbation theory in a simple way in contrast to the enormous efforts to get only lower orders with perturbative methods. The disadvantages of this numerical renormalization are the need of computer power and at present the systematic uncertainties due to a quenched approximation (that excludes fermion loops), discretization errors [2] and gauge fixing.

We look here at operators [3] $\mathcal{O} \sim \bar{q}Oq$ with quark fields q and O constructed out of Dirac operators γ_μ and covariant lattice derivatives D_ν.

2. METHOD

The renormalization method we study is a MOM-scheme using the lattice as regulator with the lattice spacing a as cut-off. Lattice operators $\mathcal{O}(a)$ and quark fields $q(a)$ are renormalized being then only a function of the scale parameter μ, via:

$$\begin{aligned} \mathcal{O}^R(\mu) &= Z_{\mathcal{O}}((a\mu)^2, g(a))\mathcal{O}(a), \\ q^R(\mu) &= Z_q^{1/2}((a\mu)^2, g(a))q(a) \end{aligned}$$

($g(a)$ is the bare lattice coupling.)

To determine the renormalization constants $Z_{\mathcal{O}}$ one imposes conditions on amputated forward quark vertex functions in momentum space with external four-momenta p:

$$\langle q(p)|\mathcal{O}(\mu)|q(p)\rangle^R_{amp} \equiv \langle q(p)|\mathcal{O}(a)|q(p)\rangle^{tree}_{amp}|_{p^2=\mu^2}$$

With a projector method, where $Tr[\Gamma_{\mathcal{O}} \times ...]$ is taken, Tr being a colour$\times$spin trace, these prescriptions give:

$$\begin{aligned} & Z_{\mathcal{O}} Z_q^{-1} Tr[\Gamma_{\mathcal{O}}\langle q(p)|\mathcal{O}(a)|q(p)\rangle_{amp}] \\ & \quad = Tr[\Gamma_{\mathcal{O}}\langle q(p)|\mathcal{O}(a)|q(p)\rangle^{tree}_{amp}]\big|_{p^2=\mu^2} \qquad (1) \end{aligned}$$

As an optimal choice for the projector, we use, written in a general form:

$$\Gamma_{\mathcal{O}} \sim \langle q(p)|\mathcal{O}(a)|q(p)\rangle^{tree}_{amp}$$

There are two definitions for Z_q: Usually one projects onto the energy-momentum part

*Talk given by H. Oelrich at Lat97, Edinburgh, U.K.

$\sin(p_\mu a)\gamma_\mu/a$ (continuum form: $\not{p}$) of the inverse propagator S^{-1} and gets:

$$Z_q = Tr\left[\frac{-i\sum_\lambda \gamma_\lambda \sin(p_\lambda a)}{(2\kappa)12\sum_\lambda \sin^2(p_\lambda a)} S^{-1}(p_\lambda a)\right]_{p^2=\mu^2}$$

(The normalization is chosen to give $Z_q = 1$ in the free case.) Another definition for Z_q can be derived from eq. (1), setting $\mathcal{O}$ equal to the conserved vector current J_μ and using $Z_{J_\mu} = 1$. But this is only compatible to the first determination of Z_q, if the component of J_μ transverse to p is taken. (This can be proven by a Ward Identity $J_\mu(p,p) = -i\partial/\partial p_\mu S^{-1}(p)$ assuming $S^{-1}(p) = iZ_q(p^2)\not{p} + B(p^2)1$, which is true in the continuum and approximately true on the lattice for not too large p^2a^2.)

3. NUMERICAL IMPLEMENTATION

In a first step Monte Carlo gluon field configurations U have to be generated and numerically fixed to the Landau gauge [4]. The lattice matrix elements in eq. (1), non-amputated, can then be calculated in those background fields:

$$\langle q(p)|\mathcal{O}(a)|q(p)\rangle \sim \sum_{i,j_1,j_2,k} \langle S_{i,j_1} O_{j_1,j_2} S_{j_2,k}\rangle_U e^{ip(x_k-x_i)} \quad (2)$$

(i, j_1,... are space-time points on the lattice.)

The propagators $S_{j_2,p} = \sum_k S_{j_2,k} e^{ipx_k}$ can be computed from a lattice Dirac equation with a momentum source, M being the fermion matrix:

$$\sum_k M_{i,k} S_{k,p} \quad = \quad e^{ipx_i} \qquad \text{(momentum source)}$$

So here the number of matrix inversions is proportional to the number of momenta. But everything else is then there: $S_{p,j_1} = \gamma_5 S^\dagger_{j_1,p}\gamma_5$ and the quark propagator $S(p) = \langle S_{p,p}\rangle_U$ with $S_{p,p} = \sum_k S_{p,k} e^{ipx_k}$ for an amputation and Z_q.

The momentum source method automatically performs all the site sums in eq. (2). Another possibility, instead of summing over j_2 (and j_1), is to choose a particular location for the operator, for example setting $j_2 = 0$. Translational invariance tells us that this will give the same expectation value after averaging over all configurations. For this method we need to solve the Dirac equation with a point source at $j_1 = j_2$ and (for extended operators where $j_1 \neq j_2$) for a small number of sources in the region around j_2:

$$\sum_k M_{i,k} S_{k,j_1} \quad = \quad \delta_{i,j_1} \qquad \text{(point source)}$$

(S_{k,j_1} has simply to be fourier transformed to S_{p,j_1} then and we have $S(p) \sim \sum_{j_1} \langle S_{p,j_1}\rangle_U$.)

For local operators and operators with a small number of derivatives the point source method would need fewer inversions, but we see from Fig.1. that relying on translational invariance to carry out the j sums leads to much larger error bars.

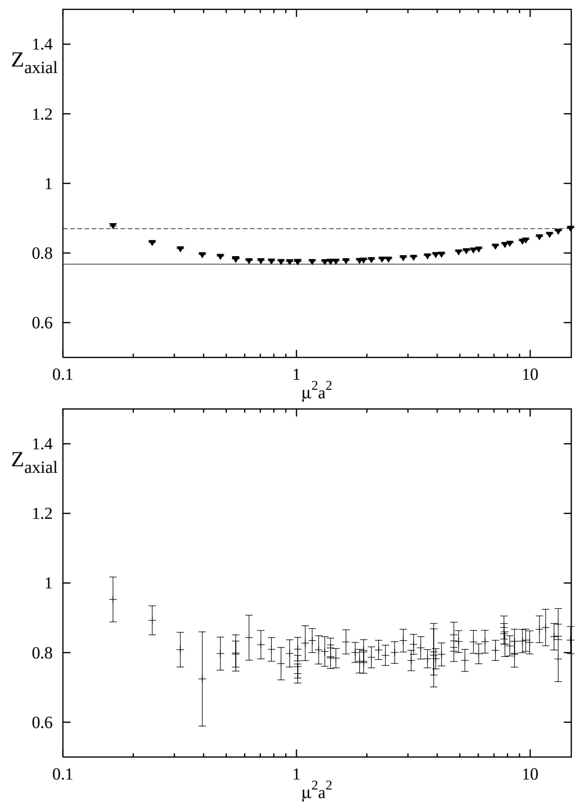

Figure 1. Z_{axial} with a momentum source method (top figure; 20 config.) and a point source method (120 config.)

4. RESULTS (WILSON FERMIONS)

Our calculations are performed on $16^3 32$ lattices at $\beta = 6.0$ for hopping parameters $\kappa = 0.1515$, 0.1530, 0.1550. If the method works for current lattices, one expects to find a high enough $\mu^2 a^2$ region where the Z factors agree with lattice per-

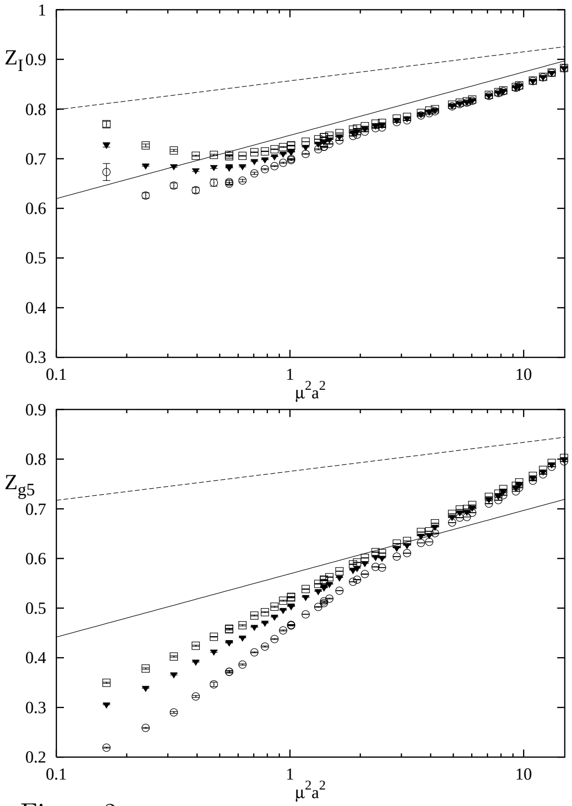

Figure 2. Z of the scalar and pseudo-scalar operator for $\kappa = 0.1515$, 0.1530, 0.1550 with lower points belonging to highest κ (lightest quark mass) are shown.

turbation theory, but are still not destroyed by discretisation effects. However the available one-loop calculations probably need to be improved for $g^2 = 6/\beta = 1$. At low $\mu^2 a^2$ non-perturbative effects and higher order perturbation theory may have a big influence as can be seen looking at the pseudo-scalar Z_{g5} (Fig.2): results for 3 quark masses indicate a non-perturbative pion pole contribution in the chiral limit. The Z factors for the operators of the moments of the structure functions, e.g. $Z_{\langle x \rangle}$ (Fig.3), seem to show also big non-perturbative or higher order perturbation theory contributions at low $\mu^2 a^2$. O(a) discretization errors are in general expected at higher $\mu^2 a^2$. However, for the local vector current (Fig.3) such errors seem to be there at all scales: it is not constant as expected from its continuum behaviour. In the O(a) improved theory [2] it is.

In a comparison with one-loop lattice perturbation theory (dashed lines in figures) [5]: $Z_{\mathcal{O}} = 1 - g^2/(16\pi^2) C_F (\gamma_{\mathcal{O}} \ln(a\mu) + B_{\mathcal{O}})$, $C_F = 4/3$ and $\gamma_{\mathcal{O}}$ being the anomalous dimension, and with tadpole improved theory (solid lines in figures) [6] we find that the improved theory fits better to the data. For the axial current (Fig.1) and the scalar operator (Fig.2) there is a good agreement. We extract $Z_{axial} = 0.78$.

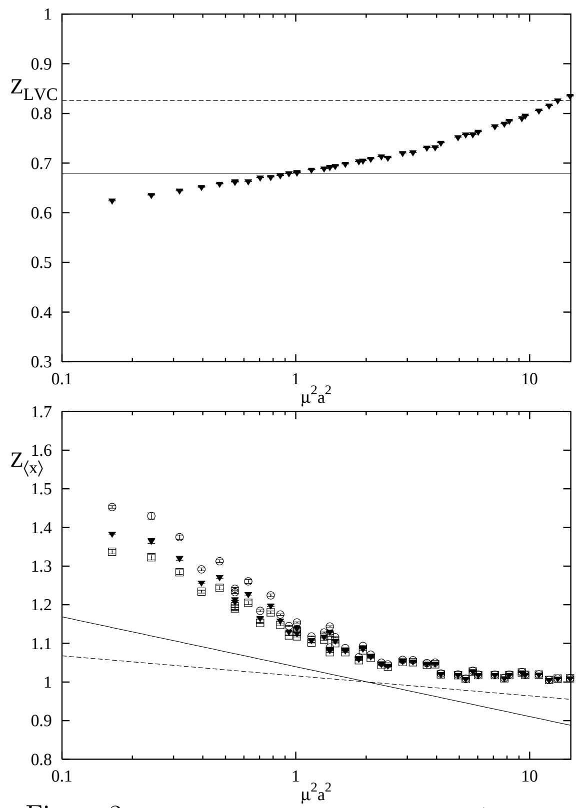

Figure 3. Z of the local vector current LVC ($\kappa = 0.1530$) and $Z_{\langle x \rangle}$ (3 κ, lower points belong to lowest κ) are shown.

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